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Collective Effects in the Horizontal Transport of Vertically Vibrated Granular Layers

P. Tegzes and T. Vicsek

Department of Atomic Physics, Eötvös University, Budapest, Puskin u 5-7, 1088 Hungary

D.C. Rapaport

Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

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Abstract

Motivated by recent advances in the investigation of fluctuation-driven ratchets and flows in excited granular media, we have carried out experimental and simulational studies to explore the horizontal transport of granular particles in a vertically vibrated system whose base has a sawtooth-shaped profile. The resulting material flow exhibits novel collective behavior, both as a function of the number of layers of particles and the driving frequency; in particular, under certain conditions, increasing the layer thickness leads to a *reversal of the current*, while the onset of transport as a function of frequency can occur either *gradually or suddenly* in a manner reminiscent of a phase transition.

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The best known and most common transport mechanisms involve gradients of external fields or chemical potentials that extend over the distance traveled by the moving objects. However, recent theoretical studies have shown that there are processes in far from equilibrium systems possessing vectorial symmetry that can bias thermal noise type fluctuations and induce macroscopic motion on the basis of purely local effects. This mechanism is expected to be essential for the operation of molecular combustion motors responsible for many kinds of biological motion; it has also been demonstrated experimentally in simple physical systems [1,2], indicating that it could lead to new technological developments such as nanoscale devices or novel types of particle separators. Motivated by both of these possibilities, as well as by interesting new results for flows in excited granular materials [3–8], we have carried out a series of experimental and simulational studies that explore the manner in which granular particles are *horizontally* transported by means of *vertical* vibration.

In the corresponding theoretical models – known as "thermal ratchets" – fluctuation-driven transport phenomena can be interpreted in terms of overdamped Brownian particles moving through a periodic but asymmetric, one-dimensional potential in the presence of nonequilibrium fluctuations [9–12]. Typically, a sawtooth-shaped potential is considered, and the nonlinear fluctuations are represented either by additional random forces or by switching between two different potentials. Collective effects occurring during the fluctuation-driven motion have also been considered [13–15], leading to a number of unusual effects that include current reversal as a function of particle density.

The investigation of an analogous transport mechanism for granular materials is an appealing idea, both conceptually and practically. By carrying out experiments – both real and numerical – on granular materials vibrated vertically by a base with a sawtooth profile, it is possible to achieve a fascinating combination of two topics of considerable current interest – ratchets and granular flows. A number of recent papers have focused on vibration-driven granular flow, and the details of the resulting convection patterns have been examined, both by direct observation [4,5,7,8] and by magnetic resonance imaging [6,16]. Granular convection has also been simulated numerically by several groups; the study most closely

related to the present work deals with the horizontal transport that occurs when the base is forced to vibrate in an asymmetric manner [17].

The present paper describes an investigation of the horizontal flow of granular material confined between two upright concentric cylinders undergoing vertical vibration. In order to induce transport, the height of the annular base between the cylinders has a periodic, piecewise-linear profile (in other words, it is sawtooth-like). We observe novel collective behavior in the resulting material flow, both as functions of the number of particle layers and the driving frequency. The most conspicuous features, for the experimental parameters used here, are that increasing the layer thickness results in a *reversal* of the current, and that the onset of transport as a function of frequency takes place in a manner analogous to a phase transition with an exponent $\beta \simeq 0.5$. Numerical simulations supporting these experimental findings are briefly described; these results also suggest the possibility of *further non-trivial transitions* as the parameters governing the system are varied.

Figure 1 shows a schematic view of the experimental apparatus. To achieve a quasi-two-dimensional system without boundaries in the direction of the expected flow the granular material is placed between two concentric glass cylinders [7]. The mean diameter of the cylinders is 10 cm, while the gap between the cylinders is either 3 or 5 mm. A ring filling the gap between the cylinders, with a sawtooth profile on its upper surface, is mounted on the base of the container. This ring is made of one of the following materials: PVC (soft), aluminum, or danamid (a hard, elastically responding synthetic material), and different sawtooth shapes are used. The entire assembly is vertically vibrated with a displacement that depends sinusoidally on time.

Two types of granular media are used in the experiments, monodisperse glass balls and quasi-ellipsoidal plastic beads (see the inset of Fig. 2). The glass balls are nearly spherical with diameter 3.3 mm $\pm$ 2%. The plastic beads have a much greater size dispersion: two of the axes are approximately equal in length and lie in the range 2.4–3.0 mm, while the third axis is 1.2–1.7 mm. As shown in the inset of Fig. 2, the size of each sawtooth is similar to that of the particles.

Provided the frequency is sufficiently large, the vertical vibration causes horizontal flow of the entire granular layer. This bulk motion is reproducible over repeated experiments. The average flow velocity is determined by tracking individual tracer particles visible through the transparent cylinder walls. In order to average out fluctuations, the particles are allowed to travel large distances; depending on the size of the fluctuations this distance is between 1.5 and 6 m (equal to 5–20 times the circumference of the system). Each point shown in the graphs is an average over 3–6 tracer particles.

Figure 2 shows the horizontal flow velocity as a function of the number of particles for various possible systems. The actual sawtooth and particle shapes are also indicated. Positive velocities are defined to be in the direction for which the left-hand edge of the sawtooth has the steeper slope (from left to the right for these cases). The vibration amplitude and frequency are $A = 2\,\mathrm{mm}$ and $f = 25\,\mathrm{Hz}$; the dimensionless acceleration $\Gamma = (2\pi f)^2 A/g$ is an important quantity for vibrated granular systems, so that here $\Gamma = 5$.

We have observed a variety of qualitatively different kinds of behavior, some of which can be interpreted by simple geometrical arguments. The most surprising phenomenon is that in certain cases the velocity changes sign; in other words, the flow *direction* depends on the layer thickness. In some cases the curves are monotonically decreasing, while others have well defined maxima. Altering the particle shape reduces the velocity and shifts the location of the maximum, but the shape of the curve remains unchanged. The only feature common to the different curves is that beyond a certain layer thickness the velocity magnitude decreases as further layers are added.

According to our studies of a simplified geometrical model the following qualitative argument can be used to explain the observed current reversal: There is an intermediate size and asymmetry of the teeth for which a single ball falling from a range of near-vertical angles bounces back to the left (negative direction) in most of the cases. This effect is enhanced by rotation, due to friction between the ball and the tooth. However, if there are many particles present, this mechanism is destroyed, and on average, the direction of the motion of particles will become positive (the "natural" direction for this geometry); this corresponds to the

usual ratcheting mechanism characterized by larger distances traveled by the particles along the smaller slope with occasional jumps over to the next valley between the teeth [9–12,22]. There is no net current for symmetric teeth, although the motion of the particles is very interesting in that situation as well [21].

We interpret the existence of maxima as follows: If only a few particles are are present, their motion is erratic, with large jumps in random directions. As the number of particles is increased, due to a inelastic collapse-like process, the particles start to move coherently, and this more ordered motion, together with the right frequency, seems to give rise to a kind of resonant behavior as far as transport is concerned. Similar maxima can also be observed in molecular motor calculations and simulations. Thick layers move more slowly because of inelastic damping.

Figure 3 shows the Γ dependence of the flow velocity for a system of 200 balls (amounting to 4 layers) for constant A. Flow occurs only above a critical acceleration $\Gamma_c \simeq 1.7$. Above this critical value the velocity appears to follow a power law

$$v(\Gamma) \propto (\Gamma - \Gamma_c)^{0.48}, \tag{1}$$

suggesting that the onset of flow resembles the kind of phase transition observed in hydrodynamic instabilities such as thermal convection [18].

In an attempt to learn about the the general nature of this class of systems, we have carried out a series of two-dimensional simulations using a molecular (or granular) dynamics approach [19,20,23]. The grains are modeled as relatively hard, rotating disks that undergo damped collisions; details of the general method and the interparticle forces appear elsewhere [24] (the damping constants here are $\gamma_s = \gamma_n = 5$), and this same system (with a flat base and without rotation) has been used in a recent study of surface excitations [25]. The base is constructed of disks similar to those representing the grains, but only 1/3 the size, positioned (with some overlap) to produce the required sawtooth profile; all the disks forming the base oscillate vertically with the appropriate amplitude and frequency. The lateral boundaries are periodic.

The simulations reveal very complex behavior that is not only in qualitative agreement with what is observed experimentally, but also suggests further directions for laboratory exploration. Horizontal transport is clearly present, but the magnitude and direction of the flow depend – often in a complex fashion – on the details of the sawtooth profile (tooth width, height and degree of asymmetry), on A and f, and also on the number of particle layers and the damping constants. Given the difficulty in constructing a model that embodies all of the actual collisional properties of the experimental system, our goal is presently limited to the qualitative reproduction of experiment.

Space permits just a single demonstration of the complex behavior. Fig. 4 shows the mean horizontal velocity (v) as a function of the number of particle layers (nl). The three curves are for systems that differ in sawtooth details, with all other parameters the same. In terms of dimensionless MD units (in which particle diameters are of order unity) the system is of width 90, $A = 1$ and $\Gamma = 2$; other details appear in the caption. Each data point is an average over 500 base oscillation cycles (the first 100 cycles of each run are excluded to allow the decay of initial transients). The results show the same kinds of behavior apparent in experiment, namely, positive or negative velocities over a range of layer depths, or a transition from negative to positive values.

The remarkable result that emerges from both experiment and simulation is that the flow direction can change as the layer thickness varies. This is entirely unexpected and requires further investigation; the only related behavior of which we are aware is the alternating current direction in a model of collectively moving interacting Brownian particles in a "flashing" [13] ratchet potential. Furthermore, the experiment described above shows a relatively sharp onset of horizontal motion, a result also obtained by simulation (not shown) for certain parameter values; in other cases, simulation reveals a more gradual onset of motion with subsequent oscillatory frequency dependence, something not yet observed in the laboratory.

In conclusion, we have investigated granular transport in a system inspired by models of molecular motors and have observed, both experimentally and numerically, that the behavior

depends in a complex manner on the parameters characterizing the system. These results ought to stimulate further research into this fascinating class of problems.

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FIGURES

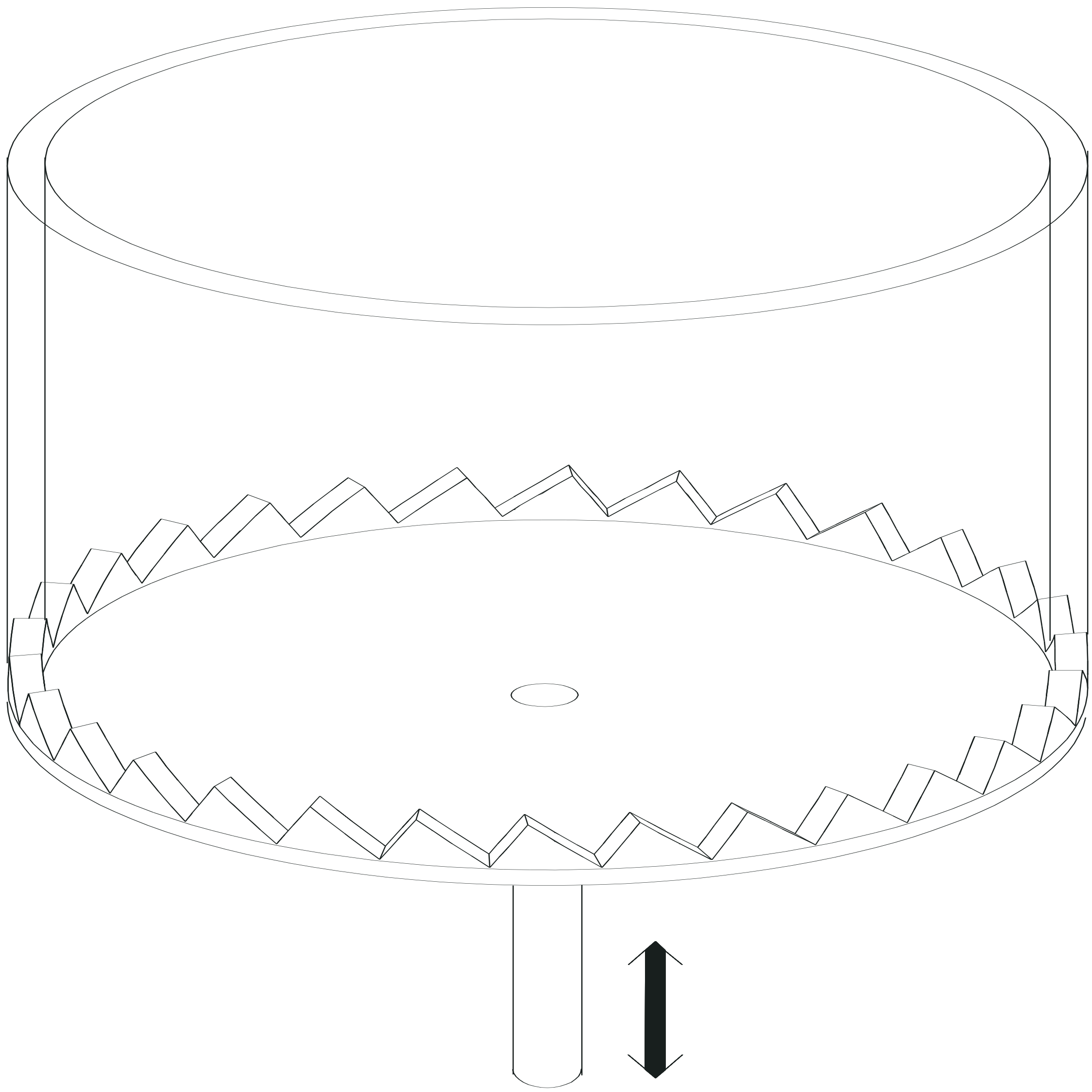

FIG. 1. Diagram of the experimental apparatus. The granular material is placed between the two glass cylinders and the whole assembly is subjected to sinusoidal vertical vibration.

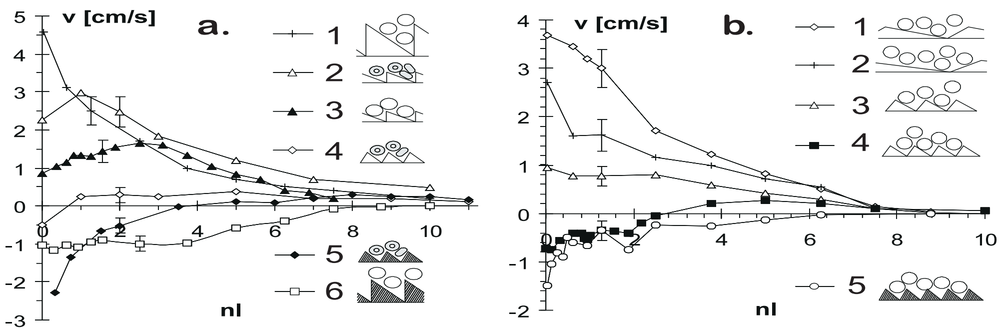

FIG. 2. Horizontal velocity as a function of the number of particles. The various curves represent measurements for various sawtooth shapes (given by horizontal projections w_1 and w_2 of the left and right parts of a tooth and its height h), materials and two kinds of particles. *a:* (1) (0 mm,10 mm,10 mm) PVC sawtooth and glass balls; (2) (0 mm,6 mm,3 mm) PVC sawtooth and plastic beads; (3) (0 mm,6 mm,3 mm) PVC sawtooth and glass balls; (4) (1 mm,3 mm,3 mm) PVC sawtooth and plastic beads; (5) (1 mm,3 mm,3 mm) hard plastic sawtooth and plastic beads; (6) (0 mm,7 mm,7 mm) hard plastic sawtooth and glass balls; *b:* All of the curves correspond to glass balls. The parameters of the sawteeth were (1) (4 mm,12 mm,3 mm) PVC; (2) (6 mm,18 mm,3 mm) PVC; (3) (1.5 mm,4.5 mm,3 mm) PVC; (4) (1.25 mm,3.75 mm,3 mm) PVC; (5) (1 mm,3 mm,3 mm) hard plastic; The amplitude and frequency are $A = 2\,\mathrm{mm}$, $f = 25\,\mathrm{Hz}$.

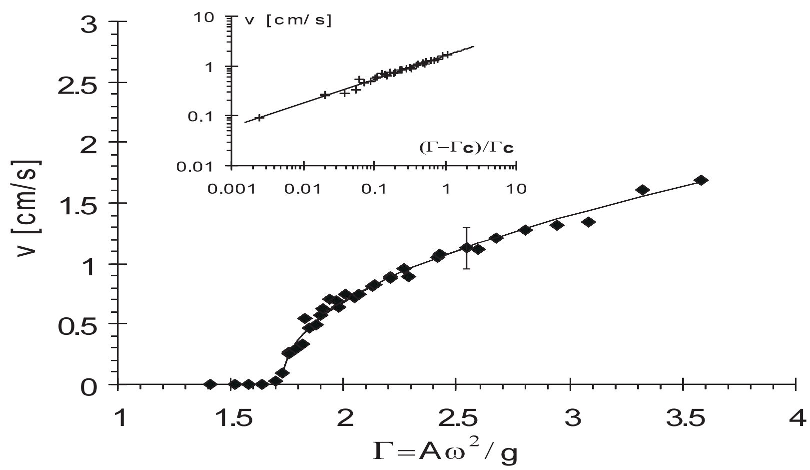

FIG. 3. Horizontal velocity v as a function of the dimensionless acceleration Γ at constant amplitude ($A = 2\,\mathrm{mm}$). The experiment is for a strongly asymmetric aluminium sawtooth ($w_1 = 0\,mm$, $w_2 = 12\,mm$, $h = 7\,mm$ and 200 glass balls. In the inset we display the data on a log-log scale for v close to the transition as a function of $(\Gamma - \Gamma_c)/\Gamma_c$ where $\Gamma_c = 1.7$. The slope of the fitted line is 0.48.

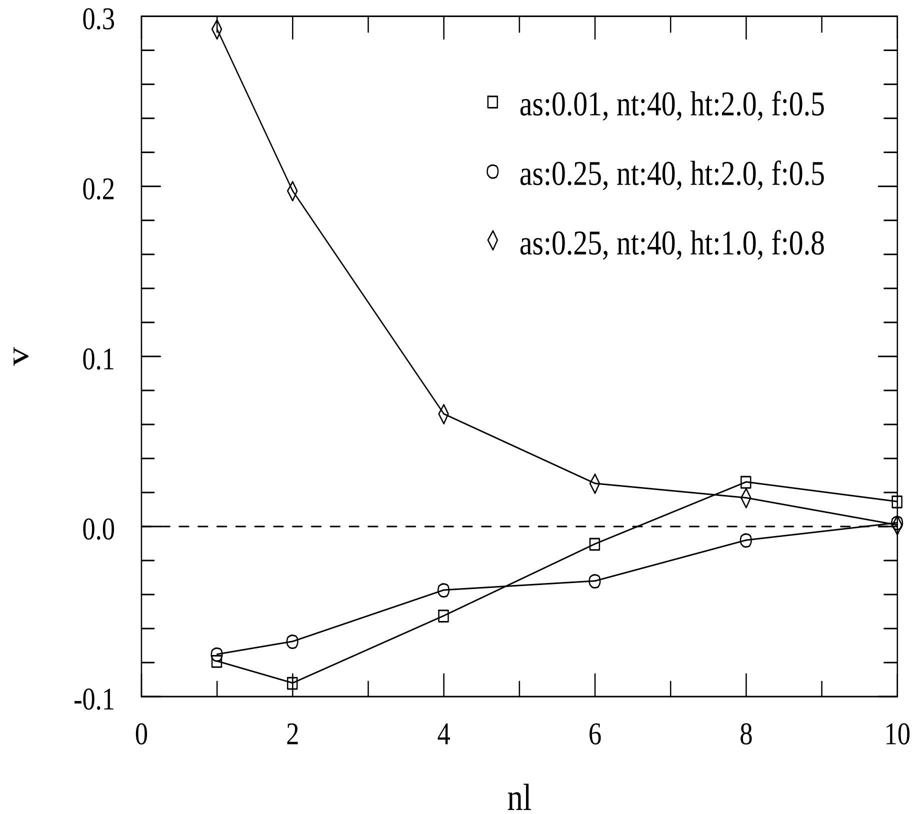

FIG. 4. Simulated horizontal velocity (v – in dimensionless units) as a function of the number of particle layers (nl); "as" denotes sawtooth asymmetry (0.01 corresponds to an almost vertical left edge, 0.25 to edges with horizontal components having a 1:3 ratio), "nt" is the number of sawteeth in the base, "ht" is the sawtooth height, "f" is the dimensionless frequency; typical spread of the velocity results, based on repeated runs with different initial conditions, is ± 0.01.